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Diffusively coupled networks

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Doctoral School Lyon, France, 11-12 April 2024

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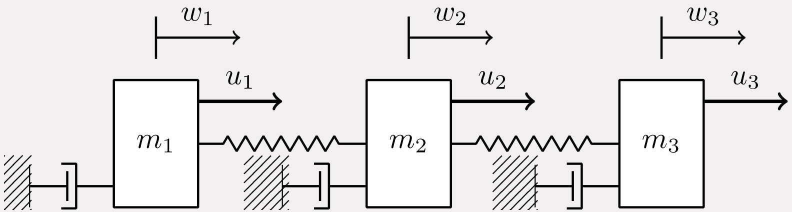
Diffusively coupled networks

Back to the basics of physical interconnections

In connecting physical systems, there is often no predetermined direction of information [1]



Example: resistor / spring connection in electrical / mechanical system:

Resistor

$$I = \frac{1}{R}(V_1 - V_2)$$

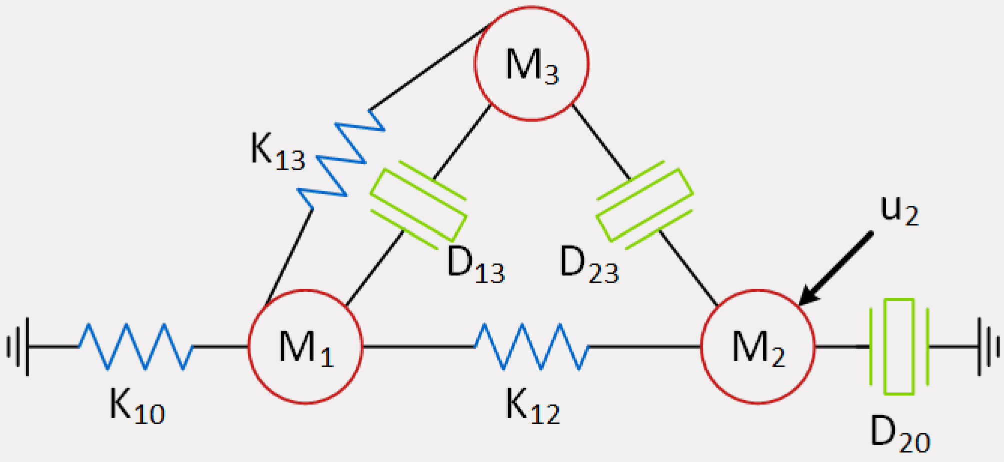
Spring

$$F = K(x_1 - x_2)$$

Difference of node signals drives the interaction: **diffusive coupling**

[1] J.C. Willems (1997,2010)

Diffusively coupled physical network

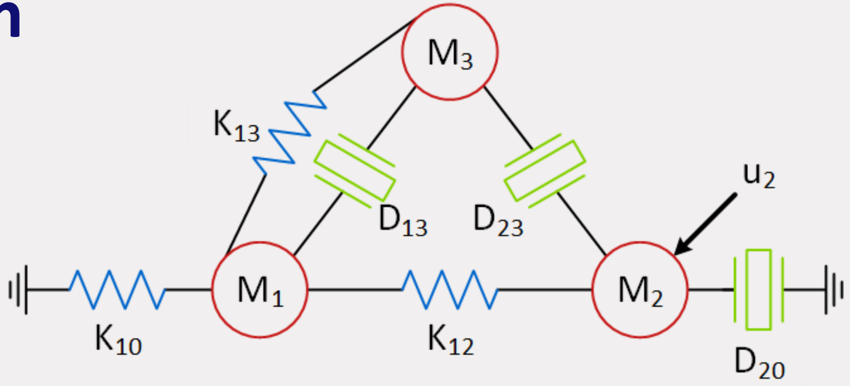


Equation for node *j*:

$$M_j\ddot{w}_j(t)+D_{j0}\dot{w}_j(t)+\sum_{k\neq j} D_{jk}(\dot{w}_j(t)-\dot{w}_k(t))+K_{j0}w_j(t)+\sum_{k\neq j} K_{jk}(w_j(t)-w_k(t))=u_j(t),$$

Mass-spring-damper system

- Masses M_j
- Springs K_{jk}
- Dampers D_{jk}
- Input u_j



$$\begin{bmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{bmatrix} \begin{bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{bmatrix} + \begin{bmatrix} 0 & & \\ & D_{20} & \\ & & 0 \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{10} & & \\ & 0 & \\ & & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$+ \begin{bmatrix} D_{13} & 0 & -D_{13} \\ 0 & D_{23} & -D_{23} \\ -D_{13} & -D_{23} & D_{13} + D_{23} \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{12} + K_{13} & -K_{12} & -K_{13} \\ -K_{12} & K_{12} & 0 \\ -K_{13} & 0 & K_{13} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 \\ u_2 \\ 0 \end{bmatrix}$$

$$[\ \underbrace{X(p)}_{diagonal} + \underbrace{Y(p)}_{Laplacian}\ ]\, w(t) = u(t) \qquad X(p), Y(p) \text{ polynomial} \qquad p = \frac{d}{dt}$$

Mass-spring-damper system

$$[\underbrace{X(p)}_{diagonal} + \underbrace{Y(p)}_{Laplacian}]\, w(t) = u(t) \qquad X(p), Y(p) \text{ polynomial}$$

$$[\underbrace{Q(p)}_{diagonal} - \underbrace{P(p)}_{hollow\&symmetric}]\, w(t) = u(t)$$

This fully fits in the earlier module representation:

$$w(t) = Gw(t) + \underbrace{Rr(t) + He(t)}_{Q^{-1}(p)u(t)}$$

with the additional condition that:

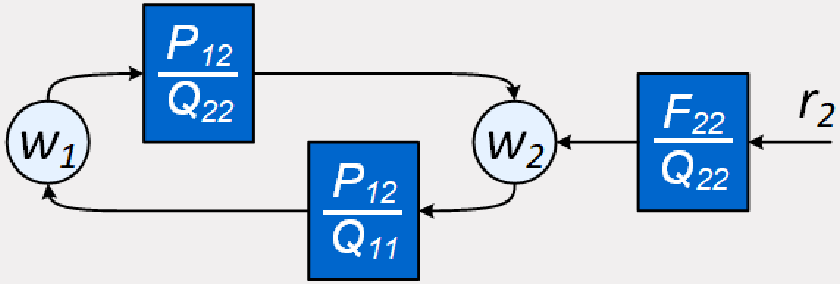
$$G(p) = Q(p)^{-1}P(p) \qquad Q(p), P(p) \text{ polynomial}$$

$P(p)$ symmetric, $Q(p)$ diagonal

TU/e

Module representation

Consequences for node interactions:



- Node interactions come in pairs of modules
- Where numerators are the same

Framework for network identification remains the same

- Symmetry can be incorporated in identifiability/identification

[1] E.M.M. Kivits et al., CDC 2019.

Polynomial representation

More attractive: stay within the polynomial domain (discrete-time now)

$$[\underbrace{Q(q^{-1})}_{diagonal} - \underbrace{P(q^{-1})}_{hollow\&symmetric}]\, w(t) = u(t)$$

$$A(q^{-1})w(t) = \underbrace{B(q^{-1})r(t) + v(t)}_{u(t)}$$

with $A(q^{-1})$ symmetric and nonmonic

i.e. $A(q^{-1}) = A_0 + A_1 q^{-1} + \cdots A_n q^{-n}$

with $A_0 \neq I$

Network identifiability[1]

New analysis, based on $T_{wr}(q)$ only (noise discarded because of algebraic loops):

$$A(q^{-1})w(t) = B(q^{-1})r(t)$$

$$\Pi(q^{-1}) \left[A(q^{-1})w(t) = B(q^{-1})r(t)\right]$$

Identifiability conditions:

- At least 1 excitation signal $r(t)$ present
- $A(q^{-1})$ and $B(q^{-1})$ left coprime
- diagonality constraint on $\left[A_0 \cdots A_n \;\; B_0 \cdots B_n\right]$
- $A(q^{-1})$ symmetric
- 1 parametric constraint in $A(q^{-1})$ or $B(q^{-1})$

- $B(q^{-1})$ present
- $\Pi(q^{-1})$ unimodular
- Π diagonal
- $\Pi = \alpha I$
- $\Pi = I$

[1] E.M.M. Kivits and PVdH, TAC 2023.

Polynomial representation - identifiability

- **Identifiability conditions** are **strongly relaxed** (compared to module framework) in terms of number of excitation signals required.

- Diffusive couplings strongly limit the **degrees of freedom** in the network model

- Identification **algorithms** are available for both full network[1] and local identification[2].

[1] E.M.M. Kivits and PVdH, TAC 2023.
[2] E.M.M. Kivits and PVdH, CDC 2022.

Summary diffusively coupled networks

- Interesting class of models, not extensively studied in identification
- Non-directed graphs
- Adhering to physical interconnections
- Framework is fit for representing **combined networks** (combining physical bi-directional links, and cyber uni-directional links)[1].

[1] E.M.M. Kivits, PhD-Thesis 2024 (to appear).

The end